DEPARTMENT OF ECONOMICS
OxCarre (Oxford Centre for the Analysis of Resource Rich Economies)

Manor Road Building, Manor Road, Oxford OX1 3UQ
Tel: +44(0)1865 281281 Fax: +44(0)1865 281163
reception@economics.ox.ac.uk www.economics.ox.ac.uk

UNIVERSITY OF OXFORD

OxCarre Research Paper 93

–

The Optimal Carbon Tax and Economic Growth: Additive versus multiplicative damages

Armon Rezai*
Vienna University of Economics and Business

Frederick van der Ploeg
OxCarre
&
Cees Withagen*
UV University Amsterdam

*OxCarre External Research Associate

Direct tel: +44(0) 1865 281281 E-mail: celia.kingham@economics.ox.ac.uk

THE OPTIMAL CARBON TAX AND ECONOMIC GROWTH:

ADDITIVE VERSUS MULTIPLICATIVE DAMAGES*

Armon Rezai†, Frederick van der Ploeg‡, §, Cees Withagen§

Abstract

In a calibrated integrated assessment model we investigate the differential impact of additive and multiplicative damages from climate change for both a socially optimal and a business-as-usual scenario in the market economy within the context of a Ramsey model of economic growth. The sources of energy are fossil fuel which is available at a cost which rises as reserves diminish and a carbon-free backstop supplied at a decreasing cost. If damages are not proportional to aggregate production output, and the economy is along a development path, the social cost of carbon and the optimal carbon tax are smaller as damages can more easily be compensated for by higher output. As a result, the economy switches later from fossil fuel to the carbon-free backstop and leaves less fossil fuel in situ. This is in contrast to a partial equilibrium analysis with damages in utility rather than in production which finds that the willingness to forsake current consumption to avoid future global warming is higher (lower) under additive damages in a growing economy if the elasticity of intertemporal substitution is smaller (bigger) than one.

Keywords: climate change, multiplicative damages, additive damages, integrated assessment models, Ramsey growth model, fossil fuel, carbon-free backstop

JEL codes: H21, Q51, Q54

10 September 2012

---

* We are grateful for the comments of the participants of the SURED conference, Ascona, June 4-7, 2012.

† Department of Socioeconomics, Vienna University of Economics and Business, Nordbergstr. 15/4/B, 1090 Vienna, Austria. Corresponding author. Email address: Armon.Rezai@wu.ac.at

‡ Department of Economics, Oxford University, Manor Road Building, Oxford OX1 3 UQ, U.K. and VU University Amsterdam.

§ Department of Economics, VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands.

1. Introduction

Climate change is the biggest externality our planet might face (see Stern (2007)). The best way to correct for that is to price carbon, either by levying a carbon tax or by having a market for carbon emission permits. The key question is what the level of the optimal carbon tax should be and what the time profile of this tax should be. Most of the literature focuses on this one externality and ignores other externalities, distorting taxes for raising public revenues, and other second-best issues. In that case, the optimal carbon tax is equal to the social cost of carbon which is the present value of global warming damages. But it matters *how* global warming damages are specified. Damages from global warming may come in many forms. Particularly in the applied integrated economic assessment models, it is customary to have damage in the production function or in the technology in general. Nordhaus (2008) is a prominent example and this approach is also chosen in Golosov et al. (2011), Stern (2007) and Tol (2002).[1] This supposes that the elasticity of substitution between damages and output is unity, which is a special assumption. Alternatively, damages can be represented through the social welfare function or, more implicitly, by imposing a ceiling.[2]

Higher global temperatures may lead to more respiratory and other diseases and thus induce lower levels of health, productivity and aggregate output. Alternatively, global warming may destroy productivity of agriculture and thus reduce aggregate output. Both of these might justify a specification with multiplicative damages. But higher global temperatures may also lead to rising sea levels and destroy part of the capital stock in which case damages should perhaps be proportional to the aggregate capital stock rather than to aggregate output. Global warming may also lead to destruction of natural habitats, e.g., the coral reefs, and to less biodiversity in which case additive damages may be more relevant. This implies an infinite elasticity of substitution between global warming damages and aggregate output. Our main objective is to investigate how the assumption of *additive* instead of *multiplicative* global warming damages affects both the level and the time profile of the carbon tax.

---

[1] Weitzman (2010) argues that one could look upon a utility function incorporating consumption and temperature as a reduced form welfare function that represents the indirect way in which temperature affects the economy's output. Hence, in this welfare function temperature should then not be interpreted as directly influencing welfare, but indirectly through damages to the potential of producing consumption commodities.

[2] First, a branch of the literature assumes catastrophic changes once the carbon concentration reaches or passes a deterministic or stochastic threshold because decay of atmospheric stops (Prieur et al. (2012)) and climate change is drastic or irreversible (e.g., Tsur and Zemel (1996) and (1998)). A problem with this approach is that usually nothing goes wrong until the ceiling is reached, whereas one would expect damages to arise already for temperatures not too far from the ceiling (see also Dullieux et al. (2011) and Chakravorty et al. (2006)). Second, damages can appear as an externality in the social welfare function (e.g., van der Ploeg and Withagen (1991, 2010, 2012a, 2012b) and John and Pecchenino (1994)). Utility from consumption can be strongly separable from the damages from climate change or not. Bretschger and Smulders (2007) argue that in the latter case with Cobb-Douglas production balanced growth is feasible (assuming away exhaustibility of non-renewables). For the case of additive separability, Stokey (1998) shows that the growth process of the economy comes to an end, if more and more output is devoted to abatement.

Integrated economic assessment models of climate change aim at integrating economics and climate change and to assess the impact economics has on climate and vice versa. This is a crucial step in the design of optimal policies to fight the potentially negative effects of climate change on economic well-being. There are various ways in which the relationship between economics and climate can and is addressed and our main aim is to investigate the effect of different types of modelling damages in a simple framework of aggregate economic growth. We are particularly interested in the modelling of the potential damage inflicted upon the economy and do not go into detail about the different ways of modelling temperature and climate change as a consequence of economic activities. Of course this is an important element, and economists are taking steps to improve the incorporation of knowledge from science into economic models, e.g. by considering the fact that some of the CO2 emissions stay in the atmosphere forever, whereas another part decays over time (e.g., Golosov et al. (2011) and Gerlagh and Liski (2011). Much of this work is based on Archer (2005) and Archer et al. (2009), but ignores the stocks of carbon in the upper and lower parts of the ocean, the delay between a rise in the stock of atmospheric CO2 and global temperature and the time-varying coefficients put forward for perhaps the first time in the path-breaking paper of Bolin and Erikkson (1958).

Dramatic differences can arise from the modelling strategy for damages in the context of climate change as has been pointed out by Weitzman (2009): "Fragility of policy to forms of disutility functions is a disturbing empirical finding because the outcomes of IAMs are then held hostage to basic structural uncertainty about the way in which high temperatures and high consumption interact". Weitzman finds that the willingness to pay, in terms of giving up present consumption, for reducing future temperature is 7 times higher in the additive case compared to the multiplicative case if the economy grows at 2% per annum and the elasticity of intertemporal substitution is set to 0.5. Furthermore, this effect disappears altogether in a stagnant economy. We show that this effect reverses in a growing economy if the elasticity of intertemporal substitution exceeds unity.

We reinvestigate Weitzman's conundrum within the context of a calibrated integrated economic assessment model of climate change based on a Ramsey model of economic growth. This has the advantage that we can allow for growth and development from an initial capital stock that is below the steady state and for the exhaustibility of fossil fuel. We analyze the effect of additive versus multiplicative production damages from climate change for both a socially optimal and a business-as-usual scenario in the market economy. The sources of energy are fossil fuel which is available at a cost which rises as reserves diminish and a carbon-free backstop supplied at a decreasing cost.

We find that, if damages are not proportional to aggregate production output and the economy is along a development path, the social cost of carbon and the optimal carbon tax are smaller than with multiplicative damages, as damages can more easily be compensated for by higher output. As a result, the economy switches later from fossil fuel to the carbon-free backstop and leaves less oil in situ. If intergenerational inequality aversion is weaker (i.e., the elasticity of intertemporal substitution is larger), we show that the optimal carbon tax is still smaller with additive damages, but that the effect is less substantial. We find that with an elasticity of intertemporal substitution of 0.5 in a fully-fledged integrated assessment model of climate change and economic growth the social cost of carbon for additive damages from global warming is about half that for multiplicative damages from global warming. In contrast, Weitzman (2009) finds for the same elasticity of intertemporal substitution that the optimal willingness to forsake current consumption to avoid future global warming is 7 times as large with additive damages if the economy grows at 2% per annum. Of course, these results are strictly speaking not comparable as Weitzman (2009) deals with damages in utility and we focus on damages in production. Still, it is important to know that Weitzman's insights do not survive once we look at *production* damages instead of *utility* damages in a fully-fledged integrated climate assessment model.

The outline of this paper is as follows. Section 2 reconsiders the main argument of Weitzman (2009) regarding additive versus multiplicative global warming damages in a partial equilibrium model and extends the results to cases where the elasticity of intertemporal substitution exceeds unity. Section 3 sets out our general equilibrium model of climate change and Ramsey economic growth with additive and multiplicative global warming damages, exogenous population growth and labour productivity growth. Carbon-free energy and exhaustible fossil fuel are perfect substitutes in production. Costs of carbon-free energy are exogenous and benefit from exogenous technical progress. Extraction costs of fossil fuel increase as fewer reserves are left in the crust of the earth. Our carbon cycle is very simple with carbon emission caused by burning fossil fuel, a constant rate of decay of atmospheric carbon, and no positive feedback loops. Section 4 gives the details of the functional forms and calibration. Section 5 gives the simulations paying particular attention to the level and the time profile of the optimal carbon tax as well as to how this tax affects the moments in time that the economy switches from using fossil fuel to the carbon-free source of energy and shows how these depend on whether global warming damages are additive or multiplicative. We also investigate the sensitivity of our main results with respect to the elasticity of substitution between energy and the capital-labour aggregate, the initial capital stock, the discount rate and the elasticity of intertemporal substitution. Section 6 concludes.

2. Revisiting Weitzman (2010)

Here we briefly review the main arguments of Weitzman (2010). Suppose that potential consumption without climate change, denoted by C^*, grows at the exogenous rate g , temperature change T is exogenous and utility is given by $U(C^*,T)=\frac{(C^*)^{1-1/\eta}}{1-1/\eta}-\alpha_A T^{1+\gamma}-\alpha_M (C^*)^{1-1/\eta}T^{1+\gamma},$ where η is the elasticity of intertemporal substitution, $\gamma > 0$ the damage parameter and $\alpha_A \geq 0$ and $\alpha_M \geq 0$ the weights given to additive and multiplicative damages, respectively. Climate damages D are a fraction of potential consumption and are implicitly defined by $\frac{\left\{\left[1-D(C^*,T)\right]C^*\right\}^{1-1/\eta}}{1-1/\eta}=U(C^*,T).$ With $\eta = 0.5$, we get multiplicative damages $D_M=\frac{\alpha_M T^{1+\gamma}}{1+\alpha_M T^{1+\gamma}}$ and additive damages $D_A=\frac{\alpha_A C^* T^{1+\gamma}}{1+\alpha_A C^* T^{1+\gamma}}.$ For high C^* and high temperature changes Weitzman thus gets higher additive than multiplicative damages.

Normalizing $C^*=1$ for $T = 0$, setting $\alpha=\alpha_A=\alpha_M$ and calibrating that 2% of current welfare-equivalent consumption is lost if the current temperature change is 2 degrees gives $U(0.98,0)=U(1,2)$ or $-\frac{1}{0.98}=-1-4\alpha,$ so $\alpha=0.0051.$ The willingness to forsake a fraction ω of current consumption to eliminate the temperature $T(t) > 0$ at some future time $t > 0$ by reducing it to $T(t) = 0$ follows from $-\frac{1}{C^*}+\frac{1}{(1-\omega)C^*}=e^{-\rho t}\left\{\alpha_A T^{1+\gamma}+\frac{\alpha_M}{e^{gt}C^*}T^{1+\gamma}\right\}.$ This yields $\omega_M=\frac{\alpha T^{1+\gamma}(t)}{e^{(g+\rho)t}+\alpha T^{1+\gamma}(t)}$ for multiplicative and $\omega_A=\frac{\alpha T^{1+\gamma}(t)}{e^{\rho t}+\alpha T^{1+\gamma}(t)}$ for additive damages, where ρ is the utility discount rate. With multiplicative damages fast-growing economies thus have a smaller willingness to forsake current consumption to avoid future climate change but not with additive damages. Setting $g=2\%, \delta=0.5\%$ and $\gamma=1$, the willingness to pay at time $t = 0$ as fraction of $C(0)=C^*(0)=1$ to avoid $T(100)=5^\circ$ by reducing it to $T(100)=0^\circ$ equals $\omega_M=1\%$ and $\omega_A=7.2\%$, respectively. This willingness to pay is thus seven times bigger with additive damages and would be even bigger if the growth rate is bigger.

To see how this result depends on the elasticity of intertemporal substitution, we extend Weitzman's analysis to the situation with $\eta > 1$. We have $\partial U(C^*,T)/\partial T=-(1+\gamma)\left[\alpha_A+\alpha_M (C^*)^{1-1/\eta}\right]T^{\gamma}$ so with high C^* and $\eta < 1$ marginal damages are smaller in the multiplicative case. However, with high C^* and $\eta > 1$ marginal damages are bigger in the multiplicative case. For example, taking $\eta \to \infty$, we get $\omega_A=\alpha_A e^{-\rho t}T^{1+\gamma}$ and $\omega_M=\alpha_M e^{-(\rho-g)t}T^{1+\gamma} > \omega_A.$ We thus establish that with high degrees of intergenerational inequality aversion the Weitzman model suggests that in a *growing* economy the willingness to forsake consumption is bigger with multiplicative than with additive damages.

In the Ramsey growth model with global warming formulated in section 3 below the growth rate is the general equilibrium outcome of consumption and saving decisions, temperature change is endogenous as it depends on past carbon emissions, and damages appear as production losses rather than as utility losses. Our objective is to find out whether Weitzman's result that the willingness to avoid global warming is in growing economies lower under additive damages if the elasticity of intertemporal substitution is less than one and lower under multiplicative damages otherwise is robust. This need not be the case, since there is no reason why additive *utility* damages, even if interpreted as reduced form damages to production, should be equivalent to additive *production* damages. In fact, with the special negative exponential damage function, logarithmic utility and Cobb-Douglas production, 100% depreciation rate, zero fossil fuel extraction costs and no backstop employed by Golosov et al. (2011) additive utility damages turn out to be equivalent to multiplicative production damages.

3. The model

Social welfare is utilitarian and consists of discounted total utility of per capita consumption. The social planner's objective is to maximize the social welfare function:

(1) $$\int_0^\infty e^{-\rho t} L(t) U(C(t)/L(t))dt.$$

Here $L(t)$ denotes population, which has an exogenous growth profile, $C(t)$ is aggregate consumption, U is the instantaneous utility function and ρ is the rate of pure time preference assumed to be a positive constant. Optimal climate policy takes place under a number of constraints in the form of a set of differential equations governing the global economy. First, output Y is produced using three inputs, man-made capital K, labour, L, and energy. Two types of energy are used. Renewables R, such as solar and wind energy, and fossil fuels like oil or natural gas, for which we use the symbol O. Besides these three inputs also the level of atmospheric carbon E plays a role, through the damages that are caused by climate change. Renewables have an exogenously varying marginal production cost $b(t)$. Extraction cost of oil is stock dependent, so that total extraction cost is given by $G(S)O$, with S the existing oil stock and G a monotonically decreasing function. We also allow for technical progress and population growth.

Production is allocated to consumption (C), net investments in man-made capital ($\dot{K}$), depreciation $\delta_K K$, with a constant rate of depreciation δ_K, and to cover the cost of resource use:

(2) $$\dot{K} = F(K, L, O+R, E, t) - G(S)O - b(t)R - C - \delta K, \quad K(0) = K_0.$$

The initial capital stock K_0 is given.

The main purpose of this paper will be to investigate the effects of differences in modelling the impact of CO2 accumulation on production. We will consider additive as well as multiplicative damages. The development over time of the oil stock is described by:

(3) $\dot{S} = -O,\ S(0) = S_0.$

The initial oil stock S_0 is given. In addition to capital and oil we have CO2 in the atmosphere E as a third state variable. Its development over time is given by:

(4) $\dot{E} = \psi O - \delta_E (E - 280),\ E(0) = E_0.$

So, it is assumed that atmospheric carbon accumulates in proportion to the use of fossil fuel. The stock of atmospheric CO2 will never be below its pre-industrial level of 280 ppmv. The initial stock is E_0. The parameter ψ is the fraction of CO2 emissions that is not immediately absorbed by available carbon sinks and δ_E is the long-term rate of natural decay of atmospheric CO2. This way of modelling CO2 accumulation is admittedly very simple and abstracts from many subtleties of modelling the carbon cycle as discussed in the seminal paper of Bolin and Erikkson (1958) and more recently in, say, Gerlagh and Liski (2012).

The current value Hamiltonian reads

$$H = LU(C/L) + \lambda\left[F(K,L,O+R,E,t) - G(S)O - b(t)R - C - \delta K\right] - \mu_S O - \mu_E[\psi O - \delta_E (E - 280)]$$

where λ, μ_S and μ_E denote the shadow value of man-made capital, the shadow value of the oil stock and the social disvalue of atmospheric CO2, respectively. Necessary conditions for an optimum are:

(5) $U' = \lambda.$

(6) $F_{O+R} \leq G + \dfrac{\mu_S + \mu_E \psi}{\lambda},\ O \geq 0,$ c.s.

(7) $F_{O+R} \leq b(t),\ R \geq 0,$ c.s.

(8) $-\dot{\lambda} = \lambda(F_K - \rho - \delta_K).$

(9) $\dot{\mu}_S = \rho \mu_S + \lambda G' O.$

(10) $\dot{\mu}_E = (\rho + \delta_E)\mu_E + \lambda F_E.$

Equations (5) and (8) lead to the usual Keynes Ramsey rule for the growth in consumption:

(11) $\dot{C} = \eta C[F_K - \delta - \rho],$

with $\eta > 0$ denoting the elasticity of intertemporal substitution. Equation (6) implies that if oil is used the marginal benefits (F_{O+R}) equal the marginal cost, which consists of the marginal extraction cost $(G(S))$, the marginal cost of extracting today rather than tomorrow (i.e., the Hotelling rent (μ_S / λ)) and the marginal social cost of CO2 ($\psi\mu_E / \lambda$). The Hotelling rent and the cost of CO2 are expressed in consumption (numeraire) terms. Equation (7) states that if renewables are used, their marginal benefits equal marginal cost. Finally, integrating equations (9) and (10) leads to:

(12) $$\mu_S(t) = -e^{\rho t}\int_t^\infty e^{-\rho s}G'(S(s))O(s)\lambda(s)ds.$$

(13) $$\mu_E(t) = -e^{(\rho+\delta_E)t}\int_t^\infty e^{-(\rho+\delta_E)s}F_E(s)/\lambda(s)ds > 0.$$

So, the shadow price of oil is the total discounted value of the future gains of leaving some oil unexploited. The social cost of CO2 is the total discounted production loss induced by a marginal instantaneous increase of CO2. Since there is only one externality in the model, a decentralized economy with a perfect set of markets but not taking into account the externality will perform in the first-best way if the tax on CO2 emissions, in money terms, is equal to the social cost of CO2:

(14) $$\tau(t) = \frac{\mu_E(t)}{\lambda(t)} = \frac{\psi\int_t^\infty e^{-(\rho+\delta_E)(s-t)}\left(\frac{-F_E(s)}{U'(C(s)/L(s))}\right)ds}{U'(C(t)/L(t))}.$$

In principle, three regimes can occur: a regime with only oil use, a regime with only use of renewables and a regime with simultaneous use. In the first regime we have equality in the first part of (6) and strict inequality in the second part. The other regimes can be characterized in a similar way. In principle each regime can occur and a priori there is no reason why a transition from one regime to any other regime should be excluded. A useful benchmark is the carbon-free economy, i.e., an economy that only uses renewables.[3] If the initial carbon stock is above the 280 level, the CO2 stock will converge to 280. The use of renewables can be derived from $F_{O+R} = b(t)$ as a function of time, capital, labor and CO2. Since we assume perfect substitutability of fossil fuel and renewables in production

[3] Van der Ploeg and Withagen (2012b) consider a similar benchmark in a model but with damages directly occurring in the welfare function (strongly separable from utility from consumption); they have neither population growth nor decay of carbon and their backstop is produced at constant marginal cost.

and we postulate that the economy is growing, there will never be simultaneous use of fossil fuel and renewables, as shown below, except possibly for a single period of time. With convex production cost of the renewable, phases with simultaneous use can occur. But we wish to highlight the transitional dynamics. The timing of the transition from fossil fuel to renewables is endogenous in our model, which is one of its special features. For the transition to take place the production cost of renewables should be smaller than the market cost of fossil fuel (assuming the imposition of an appropriate tax in the first-best case), consisting of the sum of the direct instantaneous extraction cost, the social cost of climate change and the discounted higher total extraction cost due to extracting today rather than tomorrow, the Hotelling rent.

In the carbon-free economy the accumulation of capital (2) can then be written as

$$(15) \qquad \dot{K} = \tilde{F}(K,t) - C - \delta K, \quad K(0) = K_0,$$

where $\tilde{F}$ incorporates the exogenous growth of labour, exogenous technical progress, the exogenous development of CO2 and the optimal time dependent cost of renewables. The use of renewables has been eliminated from the production function by using the fact that with positive input of renewables $F_{O+R} = b(t)$ so that we can write renewables use as a function of the other inputs. We are then back in a non-stationary Ramsey model and, if the function $\tilde{F}$ is well-behaved (for example, if the role of time in the function is equivalent to Harrod-neutral technical change, implying that all inputs can be written in intensive form), the economy will converge to balanced growth. In the absence of non-stationarity the economy converges to the golden rule. For this case van der Ploeg and Withagen (2012b) show that if an economy is developing and has a very low initial oil stock (implying that the marginal extraction cost are prohibitively high in their set up) oil will never be used, and the economy is carbon-free indefinitely. If the initial oil stock is moderate in size, it is optimal to use oil only initially and the switch to the carbon-free economy. If the economy is endowed with a lot of oil, the optimum starts with only oil as well, but will overshoot the carbon-free steady state of capital. A final stage in this case consists of simultaneous use of oil and renewables. Also, a mature economy, with an initial capital stock larger than the carbon-free steady state, will also end up in regime with simultaneous use, provided the initial oil stock is not too small. On the one hand the model that we consider here is more complicated than the model studied by van der Ploeg and Withagen, because we have population growth and time dependent cost of the backstop. Moreover, we allow for non-separability of damages in the production function. On the other hand, the assumption of ever decreasing production cost of the backstop is helpful to exclude the possibility of simultaneous use of oil and renewables eventually. In spite of the perfect substitutability of oil and renewables in production it is not evident that simultaneous use will never occur. From the necessary conditions it is clear that a condition for simultaneous use to prevail is that:

(16) $\lambda\dot{b} = \rho\mu_S + (\rho + \delta_E)\mu_E + \lambda F_E + (F_K - \rho - \delta_K)(\mu_S + \mu_E).$

The production cost of renewables is non-increasing. If the economy is still growing, implying that the marginal product of capital is large, and if the social cost of carbon is increasing, then there will not be simultaneous use. In the simulations that follow we will assume that these conditions are satisfied, but since we simulate the model in discrete time there might be simultaneous use for a single period of time. Apart from that, the transition will be smooth in the sense that there cannot occur (large) jumps in the use of total energy in production.

4. Functional forms and calibration

4.1 Functional forms

In the simulations we will use an iso-elastic utility function:

(17) $U(C/L) = \dfrac{(C/L)^{1-1/\eta} - 1}{1 - 1/\eta},$

where the elasticity of intertemporal substitution is $EIS = -\dfrac{U'}{U''C} = -\dfrac{(C/L)^{-1/\eta}}{(-1/\eta)(C/L)^{-1/\eta}} = \eta$. The elasticity of intergenerational aversion equals $1/\eta$. We employ an extraction technology of the form:

(18) $G(S) = \gamma_1 (S_0 / S)^{\gamma_2}$,

where γ_1 and γ_2 are positive constants. This specification implies that not all oil will be extracted; some oil remains in the crust of the earth. Potential production is the maximum output that can be attained if there would be no damage from climate change and is given by:

(19) $\tilde{Y} = \left[(1-\beta)\left(AK^{\alpha}(B_t L)^{1-\alpha} \right)^{1-1/\vartheta} + \beta\left(\dfrac{O+R}{\sigma}\right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}.$

So, potential output is determined by a CES production function with capital and labor appearing in a constant returns to scale Cobb-Douglas composite. The elasticity of substitution is ϑ, the production elasticity of capital in the Cobb-Douglas function is α. Potential production has constant returns to scale. A denotes total factor productivity and B_t denotes Harrod-neutral technical progress in the capital-labor aggregate. The two types of energy are perfect substitutes in production.

Actual production (Y) is potential production corrected for damages from climate change. Global warming damages are represented by:

(20) $$1-Z=1-\frac{1}{1+\zeta(E-280)^2}.$$

If damage is only multiplicative we have $Y=Z\tilde{Y}$. So, $1-Z$ is the percentage loss of potential production. If damage is only additive, we have $Y=\tilde{Y}-(1-Z)Q_0$, where Q_0 is a scaling parameter. More generally, actual production is given by:

(21) $$Y=F(K,L,O+R,Z(E),t)=(1-\mu(1-Z))\tilde{Y}-(1-\mu)(1-Z)Q_0,$$

where μ gives the share of damages due to multiplicative and additive causes, respectively. It is the key sensitivity parameter in our simulations.

4.2 Calibration

For the numerical simulation, we use a discrete-time version of the model with time running from 2010 till 2600 and time periods of 10 years. Time is measured in periods, $t=1,2,...,60$, so period 1 corresponds to 2010-2020, period 2 to 2020-2030, etc. Final time is $T=60,$ but in our discussion of the results below we emphasize the transitional dynamics in the early part of the program. In the base model we employ the following parameter values.

Preferences

The rate of pure time preference ρ is set at 10% per decade which corresponds to 0.96% per year. The rate of intertemporal substitution η in (17) is ½.

Cost of energy

Extraction costs are calibrated to give \$5/barrel for onshore oil initially. In our specification we have $G(S_0)=\gamma_1.$ So, it is the extraction cost of the first drop of oil. We take one barrel of oil to be equivalent to 1/10 ton carbon and 2.13 gigaton carbon (GtC) to be equivalent to 1 part per million by volume CO2 (ppmv). We will express all values in terms of \$trillion per ppmv. This gives approximately $\gamma_1=0.1$. The long-term cost curve of IEA (2008) gives a quadrupling of the extraction cost of oil if another 500 ppmv are extracted. Since we are considering all carbon-based energy sources (not only oil) which are cheaper to extract, we extend this to 1000 ppmv. We assume that $S_0 = 2000$ ppmv,[4] which is equivalent to 4260GtC. This gives $\gamma_2 = 2$ in (18), since we have

$$\frac{G(1000)}{G(2000)} = 4 = (\frac{2000}{2000-1000})^{\gamma_2} / (\frac{2000}{2000})^{\gamma_2} = 2^{\gamma_2}.$$

The unit cost of renewable energy is calibrated to the percentage of GDP necessary to generate all energy demand from renewables. Under a Leontief technology, with $\vartheta \to 0$, energy demand is $\sigma \tilde{Y}_t$. The cost of generating all energy carbon free is $\sigma \tilde{Y}_t b / \tilde{Y}_t = \sigma b_t$. Nordhaus (2008) assumes that it costs 5.6% of GDP to achieve this. As we are more pessimistic, we take $\sigma b_1 = 0.10$ or, with $\sigma = 0.62$ as derived below, $b_1 = 1.6$ In the future this cost falls to 1.25% of GDP (b approaches 0.204 for $t = 60$ and 0.2 at infinity). We assume that exogenous technical progress lowers this unit cost at a falling rate starting at a reduction of 1% per year. Specifically, $b_t = 0.2 + 1.76827e^{-0.1t}$. This calibration is done for a Leontief technology. We assume that for a more general technology the same parameter values can be applied.

Population growth is assumed to follow Nordhaus (2008) and UN projections. Population in 2010 is 8.6 billion people. Population growth starts at 1% per year and falls below 1% percent per decade within six decades and flattens out at 8.6 billion people: $L_t = 8.6 - 2.98e^{-0.35t}$.

The initial atmospheric *CO2 concentration* E_0 is 388 ppmv in 2010 (NOAA, 2012). We take $\psi = 1$, which boils down to assuming that all CO2 emissions go into the atmosphere initially. In reality the carbon cycle is more complex (see Archer, 2005, for the atmosphere/ocean equilibrium). In 2010 8.36 GtC are burnt from oil, natural gas and coal (BP Statistical Review of World Energy, 2011). This corresponds to an increase in CO2 concentration of 3.92 ppmv. The actual increase in atmospheric CO2 concentration in 2010 was only 2.42 ppmv (NOAA, 2010). Hence, dissipation was 1.5 ppmv. This yields a depreciation factor equal to 1.5/388=0.0038 per year (approximately δ_E =0.038 per decade). For DICE-07 this long run dissipation factor lies between 0.0025 and 0.0055 annually after all feedbacks between carbon and temperature are taken into account (see Rezai (2010)).

We assume a *damage* parameter $\zeta = 2/10^7$. Underlying this is the assumption that 0.2% of output is currently lost due to climate change (Nordhaus, 2008). At an atmospheric level of 550 ppmv, at which a temperature increase of more than 2°C is very likely according to the IPCC (2007), gives a loss of

[4] Stocks of carbon-based energy sources are notoriously hard to guesstimate. IPCC (2007) assumes in its A2 scenario that 7000 Gt CO2 (with 3.66 t CO2 per t C this equals roughly 1000ppmv) will be burnt with a rising trend this century alone. We double this number. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 Gt C (2817 ppmv) in the DICE-07.

1.4% of GDP. This is in the lower range of what is normally assumed (e.g., Tol (2002)). On the whole we assume relatively low damages, low extraction cost and high cost for renewable energy. This biases our model to give high extraction rates. Our calibrated damage function is plotted in fig. 1. The left panel reports the multiplicative function in which damages are a constant fraction of output. The right panel reports the additive damage function where damages are the same amount in monetary terms regardless of the level of output. Both functions are calibrated such that they give the same level of global warming damages for the initial levels of output and atmospheric carbon.

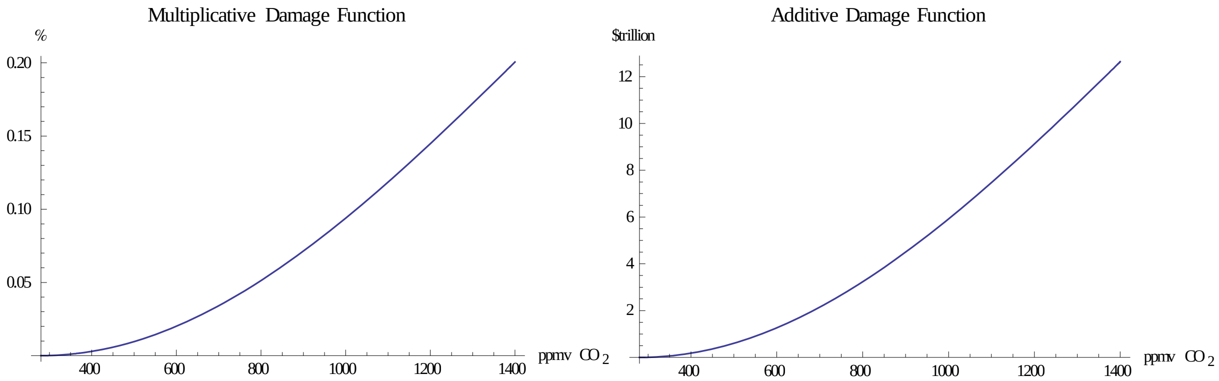


Figure 1: Damage function with $\zeta = 2/10^7$ for multiplicative and additive damages

The initial *capital stock* is set equal to 200 (US$trillion), which is taken from Rezai et al. (2012). The decay rate of capital δ_K is taken to be 0.5 per decade, which corresponds with a yearly rate of depreciation of 6.7%. It is convenient to rewrite the *potential production function* as:

$$\tilde{Y} = Y_0 \left[(1-\beta) \left(\frac{AK^{\alpha} (B_t L)^{1-\alpha}}{Y_0} \right)^{1-1/\vartheta} + \beta \left(\frac{O+R}{\sigma Y_0} \right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}} .$$

We set $\alpha = 0.35$. For ϑ we consider two alternatives: $\vartheta = 0$ (Leontief) for the benchmark run and $\vartheta = 0.5$ or what we will call the CES run. The share β is set equal to 0.07. World GDP in 2010 is 63 $trillion. Harrod-neutral technological progress occurs exogenously, $B_1 = 1$, starts out with an initial growth rate of 2% year, and stabilizes at 3 times its current level, $B_{60} = 3$. Specifically, $B_t = 3 - 2.443e^{-0.2t}$. The energy intensity of output σ is calibrated to current energy use. In the Leontief case the demand for energy (only oil initially) is $O_1 = \sigma Y_1$. With oil input amounting to 8.36GtC in 2010 and GDP in 2010 equal to 63$trillion, we get $\sigma = (8.36/2.13)/63 = 0.62$. Finally, we can back out A = 34.67. Under CES we arrive at a different value. The approach is to keep σ fixed and to use actual values for energy, labour and capital to get the output of 63$trillion.

The transversality conditions for the model are

(22) $\lim_{t\to\infty} e^{-\rho t}(\lambda_t K_t + \mu_{St} S_t + \mu_{Et} E_t) = 0$

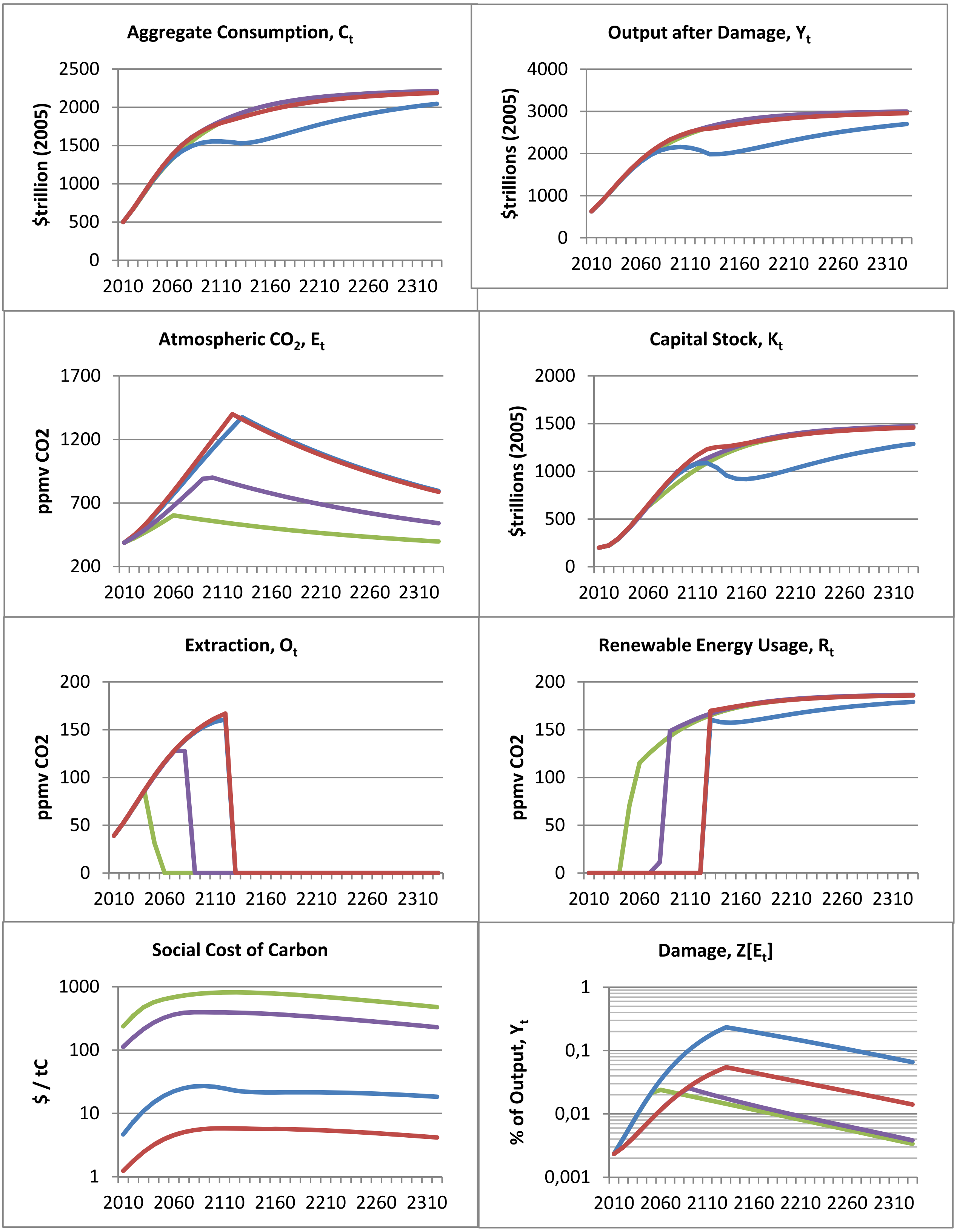
In our simulation, we solve the model for finite time and use the turnpike property to approximate the infinite-horizon problem. While the turnpike property renders terminal conditions essentially unimportant, we allow for continuation stocks to reduce the impact of the terminal condition on the transitions paths in the early periods of the program. The appendix discusses the computational implementation.

5. Policy simulation and optimization.

We report full results for eight simulations. Half of these use the Leontief production function ($\vartheta = 0$, section 5.1) and the other half employ the CES technology ($\vartheta = 0.5$, section 5.2). Moreover, we look at multiplicative and additive damages, for first-best and for business as usual, where damages are not taken into account in a "laissez-faire" economy. Finally, we perform six additional simulation runs to analyze the sensitivity of the social cost of carbon with respect to the elasticity of intertemporal substitution, the social rate of discount and the initial capital stock (section 5.3).

5.1 Leontief technology.

We start with the *first-best* outcomes of the economy with a Leontief technology. We first compare additive and multiplicative damages, which correspond to the purple and green lines in fig. 2. The first, second and fourth panels show aggregate consumption, total net output and the aggregate capital stock for the two cases. Over the entire period of time under consideration output net of damage is monotonically increasing. Moreover, net output is almost the same in both situations. The same holds for the capital stock and for consumption. This implies that total welfare is hardly affected by whether the damage function is additive or multiplicative. The difference is essentially only in the use of fossil fuel and the timing of the transition to renewables. The economy's endowments and technological change allow the economy to grow. So, if the economy with the multiplicative damage specification would use the same rate of fossil fuel, damages in terms of loss of production would be much higher over time. Therefore, the economy with multiplicative damages uses less fossil fuel, leaves more fossil fuel unexploited and makes the transition to renewables at an earlier stage. The differences are considerable. With multiplicative damages 1720 ppmv is left in the crust in the earth, and the transition to renewables takes place as soon as 2050. For the additive case much less fossil fuel is left in situ, i.e., 1280 ppmv, and renewables are phased in much later, i.e., in 2090. For the first years until 2040 no differences in extraction rates can be observed. In both cases the social cost of carbon is

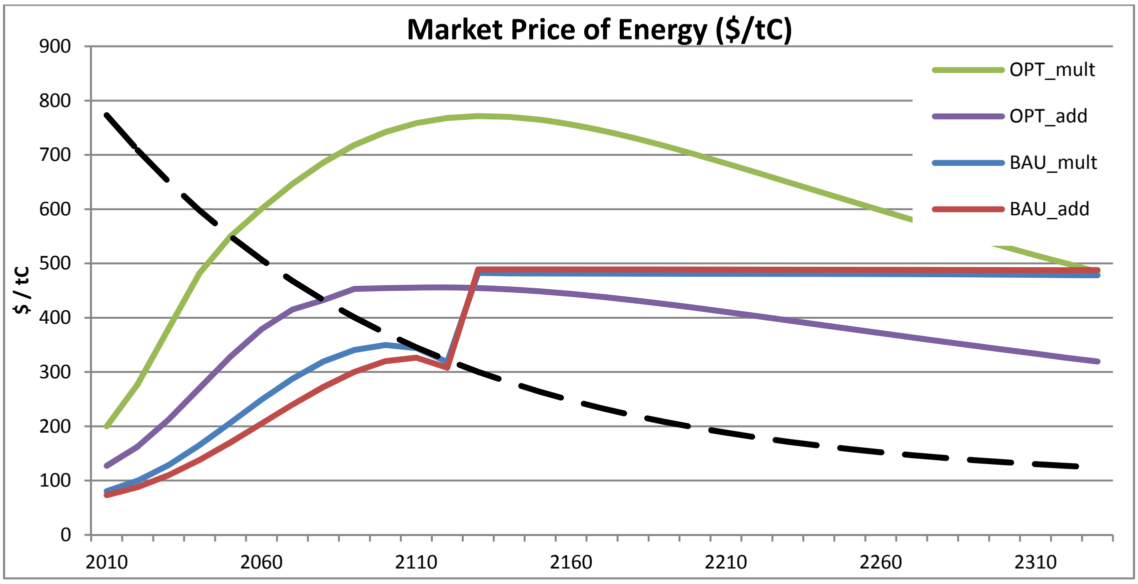


Key: Social optima: multiplicative damages (green); additive damages (purple)
Business as usual scenarios: multiplicative damages (blue), additive damages (red)

Figure 2: Simulations with Leontief production technology

inverted U-shaped, which results from the fact that CO2 emissions first rise and then come to an end. The location of the two curves is, of course, different. In the multiplicative case the social cost of carbon starts at a level of 237 $/tC and reaches a maximum of 815 $/tC in the year 2140. For the additive case we start with a much smaller social cost of 110 $/tC and the maximum is reached at about 400 $/tC in the year 2090. Weitzman (2010) finds that with additive damages the willingness to sacrifice current consumption to avoid future global warming is seven times higher than with multiplicative damages (see section 2). In contrast, we find that additive damages leads to about half the social cost of carbon at each point of time compared to multiplicative damages. However, this stark difference should serve as a reminder that the additive utility damages used in Weitzman and the additive production damages used in our model are not comparable. Weitzman's number applies to the willingness to pay to avoid any change in temperature, whereas the social cost of carbon of our simulations reports the willingness to pay to avoid a marginal increase in atmospheric carbon. All we can say is that within a fully specified integrated climate assessment model additive damages lead to a less ambitious climate policy.

The energy price is depicted in fig. 3 as well as the cost of renewable energy. The energy price in the first-best outcome is the shadow price of fossil fuel, which consists of the marginal extraction cost, the Hotelling rent (i.e., the present discounted sum of all extraction cost savings due to a higher fossil fuel stock) and the social cost of carbon. The shadow price of fossil fuel increases initially because all three components of the social cost increase.



Key: Social optima: multiplicative (green); additive (purple)
Business-as-Usual scenario: multiplicative (blue); additive (red)
Market price of energy includes optimal carbon tax in the social optima.

Figure 3: The market price of energy

Once these costs exceed the marginal cost of renewables, renewables take over. From then on the marginal extraction cost of fossil fuel is constant, the Hotelling rent is zero as well, because some oil is left in the crust of the earth, but no extraction takes place. However, the social cost of CO2 continues to rise for some time, because decay is limited and consumption is increasing, yielding smaller marginal utility of consumption and thus a higher social cost of CO2, expressed in the numeraire (see equation (14)). However, after some point of time decay of atmospheric CO2 will dominate the decrease in marginal utility of consumption and the social cost of carbon will start to fall. In theory this may eventually lead to taking fossil fuel into exploitation again, but, as the figure indicates, the time horizon that we consider is too short to make this optimal.

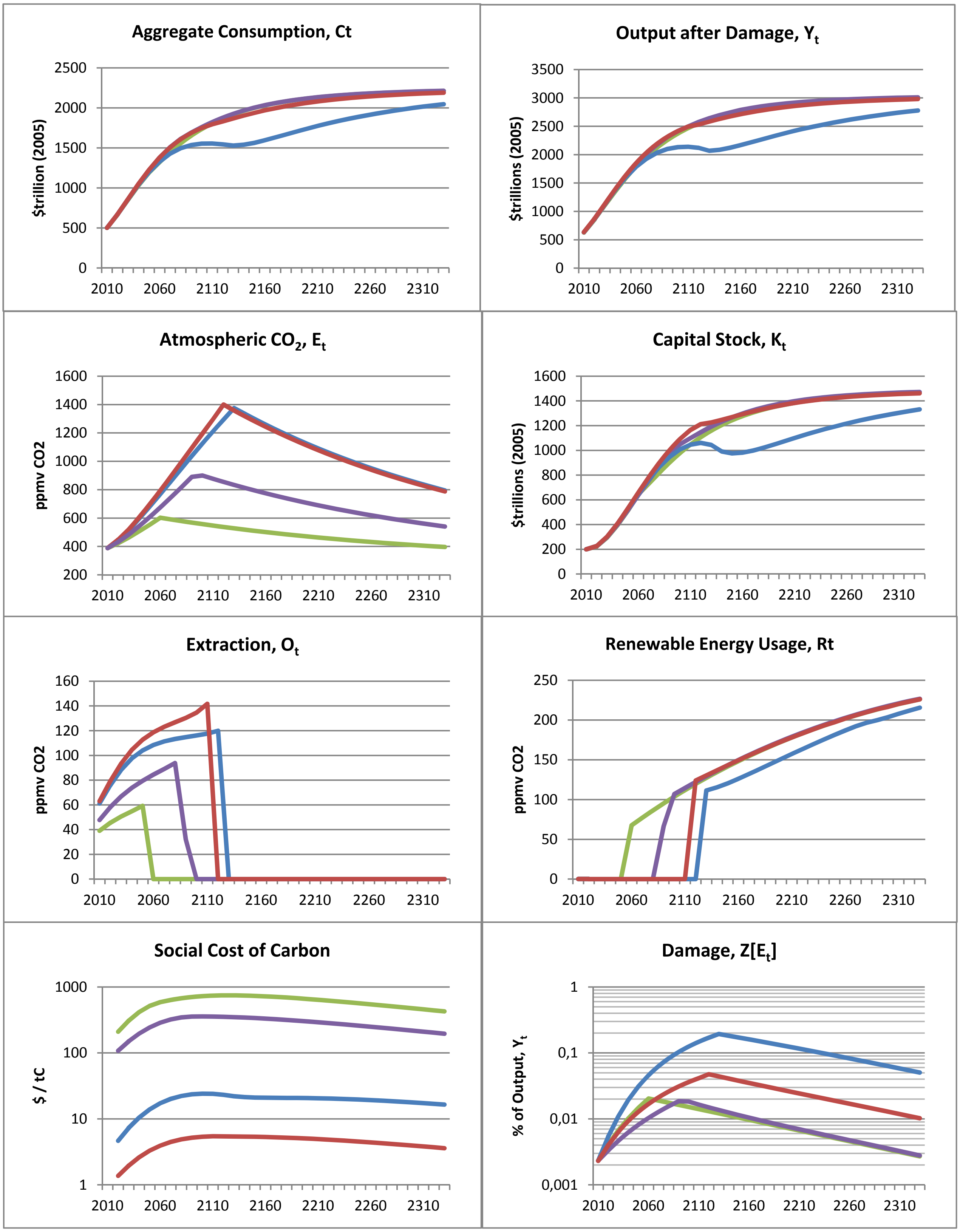
The additive and the multiplicative cases in the *business-as-usual* scenario correspond to the red and blue lines plotted in fig. 2, respectively. We find striking differences with the socially optimal outcomes. Since the CO2 stock is hardly be affected by the emissions of individual agents, they use almost the same amount of fossil fuel for approximately the same period of time, till about 2130. Extraction costs and the Hotelling rent dominate as virtually all of the negative costs of carbon emissions are not taken into account. At the end of the period a little bit more fossil fuel is used in the economy with the additive damages than in the one with multiplicative damages. Under additive damages 632 ppmv of fossil fuel remain in situ compared to 647 ppmv under multiplicative damages. But, as is to be expected, damages to production are much higher in the multiplicative case, and therefore consumption will be lower. This becomes particularly manifest after 2090. In the very long run, when atmospheric CO2 is subject to decay, (slow) convergence of the two paths takes place. A particular feature of the simulations is that in the multiplicative case capital is decreasing for several decades immediately after the economy stops using oil. We also see that, in spite of higher input of fossil fuel, net output decreases over several decades preceding the transition to renewables. This indicates that in the fossil fuel case capital is over-accumulated, which is then corrected in the renewables-only phase. In the BAU scenario the social cost of carbon does not vanish, because each individual agent is aware of the fact that she is responsible for 1% of total emissions, and therefore for damages (see appendix for details on the BAU). We also observe that much more oil is used in BAU and that the transition to renewables takes place much later (see also van der Ploeg and Withagen (2012b), who find a similar result).

Finally, although not our primary focus, it is interesting to see how the market outcome differs from the first-best outcome. Output, consumption and capital accumulation take place at very similar levels for the first-best solutions and the “laissez-faire” with additive damages. The reason for this is mainly that in a growing economy net output is not much affected by temperature changes if affordable mitigation is available (first-best scenarios) or damages are low (additive BAU scenario). Under multiplicative damages of global warming, the impacts of the climate externality are large enough to drastically change accumulation paths. This is also reflected in total welfare which is about 10% lower in the BAU case with multiplicative damages compared to the three other cases.

Sinn (2007), Foley (2009), and Stern (2010) point out that “laissez-faire” or a no-policy scenario leads to an inefficient allocation of resources, because economic decision-makers do not recognize the deleterious effects of greenhouse gas (GHG) emissions. Private and social cost calculations diverge; agents overvalue the returns to conventional capital stock and undervalue the investments in green energy sources. Imperfect price signals (λ, μ_S and μ_E) induce excess oil extraction and capital accumulation, leading to high climate damages over the time horizon. The inefficiency of this business-as-usual (BAU) scenario consists in low consumption to allow accumulation in early periods of the program leading to low consumption in the future due to high climate damages. As agents overvalue the return to conventional capital (a high perceived interest rate), they want to accumulate too large amounts of capital (relative to first-best). High damages lower net output and the returns to capital, providing a check on the accumulation aspiration. This reasoning applies to any “laissez-faire” equilibrium irrespective of the damage specification. While the amount of carbon extracted in both laissez-faire scenarios are roughly the same, only damages under the multiplicative specification are large enough to lower factor returns sufficiently to induce a decumulation of capital. Once the economy switches to renewable energy and stocks of atmospheric carbon recede, the return to capital and the interest rate increase, leading to a resumption of growth. Rezai et al. (2012) discuss this mechanism in more detail and demonstrate the important implications of this inefficiency for the debate on the (opportunity) cost of climate change in a simple model of Leontief production technology and unlimited stocks of oil.

5.2. CES technology

With a CES technology the substitution possibilities between the capital-labour aggregate on the one hand and energy on the other hand are feasible, in contrast with the Leontief technology. This implies that energy demand is more sensitive to relative price changes. In the previous section we have already discussed the relationship between scenarios in economies with additive and multiplicative damages. The outcomes for the case of a CES production function are presented in fig. 4 and inspection confirms that the qualitative differences between additive and multiplicative production damages are unaffected. For the sake of brevity, we concentrate here on the differences brought about by allowing for a higher degree of substitutability. The patterns of optimal capital accumulation and consumption are hardly affected. The use of fossil fuels is to a large extent determined by the social cost of damages. With a higher degree of substitutability far less fossil fuel is used initially in the optimum. This also holds in the market economy. Therefore, substitutability helps. We also see that renewables are phased in much more gradually after extraction of fossil fuels has come to an end. This holds for



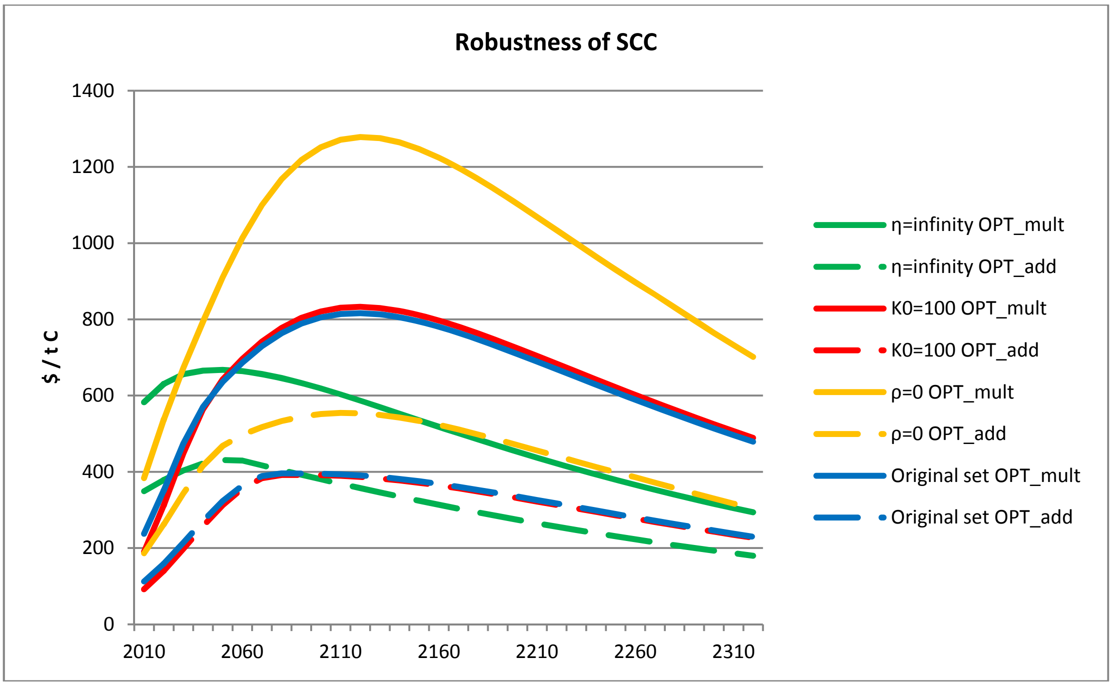
Key: Social optima: multiplicative damages (green); additive damages (purple)
Business as usual scenarios: multiplicative damages (blue), additive damages (red)

Figure 4: Simulation results with CES production technology

the first best as well as for the market economy. For the market however, these damage costs play only a minor role, implying that with better substitution possibilities more fossil fuel is used initially but for a shorter period of time, until 2100 instead of 2130, leaving about the same amount in the crust of the earth when extraction stops.

5.3. Robustness of the optimal social cost of carbon

Further robustness exercises for first-best with the Leontief technologies with respect to some other key parameters are shown in fig. 5. We focus on the social cost of carbon, so omit detailed time paths of aggregation consumption, output, the capital stock, oil use and production, renewables use and the stock of atmospheric CO2. We note from fig. 5 that starting with half the initial capital stock hardly affects the social cost of carbon irrespective of whether damages are additive or multiplicative. This result arises despite the economy being initially on a faster growth path.



Key: benchmark (blue); infinite elasticity of intertemporal substitution (green); half the initial capital stock (red); zero pure rate of social time preference (yellow) multiplicative production damages (solid); additive production damages (dashed).

Figure 5: Sensitivity analysis for the time paths of the social cost of carbon

A much lower social rate of discount leads to a much more ambitious climate policy with a much higher social cost of carbon, earlier phasing in of renewables and more fossil fuel is left in situ. Climate policy is again more ambitious with multiplicative than with additive damages.

A higher elasticity of intertemporal substitution corresponds to a lower coefficient of intergenerational inequality aversion (which was set to 2 in fig. 2-4). Fig. 5 plots the utilitarian case of zero intergenerational inequality aversion. This implies that the carbon tax hurts earlier generations much more than later generations, both in the additive and in the multiplicative case. The social planner is relatively more concerned with fighting global warming than with avoiding big differences in consumption of different generations.

6. Conclusions

How global warming damages are modelled and calibrated matters for the social cost of carbon and climate policy. Weitzman (2009) finds that the willingness to pay, in terms of giving up present consumption, for reducing future temperature is 7 times higher with additive global warming utility damages compared to multiplicative utility damages if the growth rate is 2% per annum and the elasticity of intertemporal substitution is 0.5. This effect reverses in Weitzman's analysis if the elasticity of intertemporal substitution exceeds unity. The effect disappears completely in stagnant economies. We have reinvestigated the robustness of these qualitative partial equilibrium insights within the context of a calibrated integrated climate assessment model based on a Ramsey model of economic growth with oil and carbon-free renewables as backstop and oil extraction costs that increases as fewer reserves are left in situ. In contrast to Weitzman (2009), we have focused on global warming damages to production rather than to utility and on the policy question whether climate policy is more or less aggressive with additive instead of multiplicative damages.

We find that the optimal carbon tax is lower with additive damages than with multiplicative damages and that this ranking is independent on how tough society finds it to substitute present for future consumption. So, if the coefficient of intergenerational aversion is 2, the social cost of carbon is about half as large for additive damages as for multiplicative damages in our general equilibrium model. This compares with 7 times as large in Weitzman's analysis, so that we cannot infer anything from partial equilibrium analysis of damages in utility to general equilibrium analysis of damages to production. The social cost of carbon and the optimal carbon tax are in our integrated climate assessment model smaller with additive damages as they can more easily be compensated for by higher output. Furthermore, the economy switches later from fossil fuel to the carbon-free backstop and leaves less oil in situ.

Our integrated assessment model also indicates that a higher elasticity of intertemporal substitution and a lower social rate of discount lead to a higher optimal carbon tax and a quicker phasing in of renewables and more fossil fuel left in the crust of the earth, less so under additive than multiplicative global warming damages. A higher elasticity of intertemporal substitution corresponds to a lower coefficient of intergenerational inequality aversion. Since society is more concerned with fighting global warming than with avoiding big differences in consumption of different generations, the carbon tax is borne much more by earlier generations than by later generations, both in the additive and in the multiplicative case.

In our Ramsey growth model with global warming the growth rate is the general equilibrium outcome of consumption and saving decisions, temperature change is endogenous as it depends on past carbon emissions, and damages appear as production losses rather than as utility losses. We conclude that one must be careful to generalize the qualitative insights derived from Weitzman's partial equilibrium analysis for utility damages to the effects of production damages in a general equilibrium framework.

The future developments in the productive capacity of the economy are important determinants of the social cost of carbon and the optimal carbon tax. Future prices of clean and dirty sources of energy and their necessity in the general production process heavily influence relative prices and the allocation of resources today. We examined the effects of variations in the substitutability between energy and conventional capital through a CES production function with a fixed rate of technological progress. Recent contributions by Acemoglu et al. (2012) and Mattauch et al. (2012) highlight the importance of learning and lock-in effects by making the rate of technical progress as endogenous. It is possible to use the empirical estimates of the determinants of growth rates in total factor and energy productivities given in Hassler et al. (2011) in our model of economic growth and climate change. This will allow much more substitution possibilities between energy and the capital-labour aggregate in the long run than in the short run. The logic of directed technical change suggests that it is more important to have substantial R&D subsidies for green technology to kick-start green innovation and fight global warming.

References

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012): "The Environment and Directed Technical Change", *American Economic Review*, 102(1): 131-166.

Archer, D. (2005): "The fate of fossil fuel CO2 in geologic time", *Journal of Geophysical Research* 110, C09S05.

Archer, D. Eby, M., Brovkin, V., Ridgwell, A., Cao, L., Mikolajewicz, U., Caldeira,, K., Matsumoto, K., Munhoven, G., Montenegro, A. and Tokos, K. (2009): "Atmospheric lifetime of fossil fuel carbon dioxide", *Annual Review of Earth and Planetary Sciences* 37, pp. 117-134.

Bolin, B. and Eriksson, B. (1958): "Changes in the carbon dioxide content of the atmosphere and sea due to fossil fuel combustion", in B. bolin (ed.), *The Atmosphere and the Sea in Motion: Scientific Contributions to the Rossby Memorial Volume*. New York, Rockefeller Institute Press, pp. 130–142.

Bretschger, L. and Smulders, S. (2007): "Sustainable resource use and economic dynamics", *Environmental and Resource Economics* 36, pp. 1-13

Chakravorty, U., Magne, B. and Moreaux, M. (2006). A Hotelling model with a ceiling on the stock of pollution, *Journal of Economic Dynamics and Control* 30, pp. 2875-2904.

Dullieux, R., Ragot, L. and Schubert, K. (2011): “Carbon tax and OPEC’s rents under a ceiling constraint”, *Scandinavian Journal of Economics* 113, pp. 798-824.

Gerlagh, R. and Liski, M. (2012): “Carbon prices for the next thousand years”, *CESifo Working paper* 3855.

Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A. (2011): “Optimal taxes on fossil fuel in general equilibrium”, *NBER Working paper* 17348.

Hassler, J., Krusell, P. and Olovsson, C. (2011), “Energy-Saving Technical Change", mimeo, Institute of International Economic Studies, Stockholm University.

IEA (2008) *World Energy Outlook 2008*, http://www.iea.org/textbase/nppdf/free/2008/weo2008.pdf.

John, A. and Pechenino, R. (1994): “An overlapping generations model of growth and the environment”, *Economic Journal* 104, pp. 1393-1410.

Karp, L. and Tsur, Y. (2011): “Time perspective and climate change policy”, *Journal of Environmental Economics and Management* 62, pp. 1-14.

Mattauch, L. Creutzig, F. and Edenhofer, O. (2012): “Avoiding Carbon Lock-In: Policy Options for Advancing Structural Change”, *Climatecon Working Paper Series* No. 1-2012.

NOAA Earth System Research Laboratory (2010): ”Annual Mean Growth Rate for Mauna Loa, Hawaii”, *Trends in Atmospheric Carbon Dioxide*. Retrieved 25 August 2012.

Nordhaus, W. (2008). *A Question of Balance: Economic Models of Climate Change.* Yale University Press, New Haven, CT.

Nordhaus, W.D. and Yang , Z.(1996): “A regional dynamic general-equilibrium model of alternative climate-change strategies”, *American Economic Review 86:4*, pp. 741-765.

Ono, T. (2003): “Environmental tax policy in a model of growth cycles”, *Economic Theory* 22, pp. 141-168.

Ploeg, F. van der and Withagen, C. (1991): “Pollution control and the Ramsey problem”, *Environmental and Resource Economics* 1, pp. 215-236.

Ploeg, F. van der and Withagen, C. (2010): “Is there really a Green Paradox?”, Research Paper 35, Oxcarre, University of Oxford, Oxford, U.K., forthcoming in *Journal of Environmental Economics and Management.*

Ploeg, F. van der and Withagen, C. (2012a): “Too much coal, too little oil”, *Journal of Public Economics 96,* pp. 62-77.

Ploeg, F. van der and Withagen, C. (2012b): “Growth, renewables and the optimal carbon tax”, *Research Paper* 55, Oxcarre, University of Oxford, Oxford, U.K.

Prieur, F., Tidbal, M. and Withagen, C. (2012): “Optimal emission-extraction policy in a world of scarcity and irreversibility”, *Research Paper* 77 Oxcarre, University of Oxford, Oxford, U.K..

Rezai, A. (2010). Recast the DICE and its Policy Implications, *Macroeconomic Dynamics*, 14 (S2) 275-289

Rezai, A. (2011): “The opportunity cost of climate policy: A question of reference”, *Scandinavian Journal of Economics* 113, pp. 885–903.

Rezai, A., Foley,D. and Taylor, L. (2012): “Global warming and economic externalities”, *Economic Theory* 49, pp. 329-351.

Shiell, L. and Lyssenko, N. (2008): „Computing business-as-usual with a representative agent and a pollution externality”, *Journal of Economic Dynamics and Control* 32, pp. 1543-1568.

Stern, N. (2007), *The economics of climate change: The Stern review*, Cambridge University Press.

Stokey, N. (1998): “Are there limits to growth?”, *International Economic Review* 39, pp. 1-31.
Tol, R. (2002): “Estimates of the damage costs of climate change”, *Environmental and Resource Economics* 21, pp. 135-160.
Tsur, Y. and Zemel, A. (1996): “Accounting for global warming risks: Resource management under event uncertainty”, *Journal of Economic Dynamics and Control* 20, pp 1289-1305.
Tsur, Y. and Zemel, A. (1998): “Pollution control in an uncertain environment”, *Journal of Economic Dynamics \& Control* 22, pp.967-975.
Tsur, Y. and Withagen, C. (2011): “Preparing for catastrophic climate change”, *mimeo*, Hebrew University Jerusalem.
Weitzman, M. (2009): “Additive damages, fat tailed climate dynamics and uncertain discounting”, *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 3, 2009-39.
Weitzman, M. (2010): “What is the “damage function” for global warming- and what difference does it make?”, *Climate Change Economics* 1, pp. 57-69.

Appendix: Computational implementation

We use the computer program GAMS and its optimization solver CONOPT3 to solve the model numerically. The social planner solution, OPT, in which the externality is taken into account fit the program structure readily. To solve the “laissez-faire” BAU equilibrium paths, we adopt the iterative approach of Nordhaus and Yang (1996) and Shiell and Lyssenko (2008). Rezai (2011) provides a detailed summary. To approximate the externality scenario, the aggregate economy is divided into *N* dynasties. Each dynasty has *1/N*th of the initial endowments and chooses consumption, investment and energy use in order to maximize the discounted total utility of per capita consumption. The dynasties understand the contribution of their own emissions to the climate change, but take the emissions of others as given. The climate dynamics and the resulting damages are affected by the decisions of all dynasties. This constitutes the negative externality.

It might seem easier to simply assume that there is one dynasty that ignores the externality. Such a problem is not an optimization problem. The CONOPT3 solver of GAMS is very powerful in solving maximization problems and it is more efficient to adopt an iterative routine than to attempt solving the equilibrium conditions directly. Given a technological specification, the computation of all four scenarios takes less than one minute.

To introduce this approximate externality, we make the following adjustments to the initial stocks $K(0) = K_0 / N, S(0) = S_0 / N$ and $L(0) = L_0 / N$. All production and cost functions are homogeneous of degree 1 and therefore invariant to *N*. The introduction of the pollution externality only requires a modification of the transition equation of atmospheric carbon to include emissions regarded as exogenous by each dynasty:

$$\dot{E} = O + \delta_E (E - 3280) + Exg.$$

In the BAU scenario all dynasties essentially play a dynamic non-cooperative game, which leads to a Nash equilibrium in which each agent forecasts the path of emissions correctly and all agents take the same decisions. As all dynasties are identical, equilibrium requires $Exg = (N-1)O.$ Under BAU the decision maker only adjusts her controls to take into account the damages of her own emissions (i.e., 1/*N*th of the greenhouse gas externality). If $N = 1$ the externality is internalized and we obtain the social optimum, OPT. As $N \to \infty$, we obtain the "laissez-faire" outcome characterized in section 2.

Following Rezai (2011), the numerical routine starts by setting the time path of emissions exogenous to the dynasty's optimization, $Exg(t)$, at an arbitrary (but informed) level. GAMS solves for the representative dynasty's welfare-maximizing investment, consumption, and energy use choices conditional on this level of exogenous emissions. $(N-1)$ times the dynasty's emission trajectory implied by these choices, O, defines the time profile of exogenous emissions in the next iteration step. The routine is repeated and $Exg(t)$ updated until the difference in the time profiles between iterations meets a pre-defined stopping criterion. In the equilibria reported in the text, the iteration stops if the deviation at each time period is at most 0.001%.

We set $N = 100$ to account for the fact that in the present world economy, the externality in the market of GHG emissions is already internalized to a very small extent through the imposition of carbon taxes or tradable emission permits and non-market *regulation* (e.g. through the Kyoto Protocol or the establishment of the European Union Emission Trading Scheme). In our BAU simulations, the dynastic planner takes into account less than 1% of global emissions.